

THE DEMAND FOR ENERGY IN MINORITY HOUSEHOLDS

A THESIS

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ABSTRACT

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The primary purpose of this thesis is to examine the price and income elasticity measures obtained from imposing alternative functional forms on the same set of data.

The study cites econometric literature on energy demand to isolate the specification issues that have been addressed by other authors, and an empirical analysis of some of the issues identified through the literature. The following functional forms were specified and estimated in this study: 1) linear model, 2) double-log model, 3) reciprocal (inverse) model, 4) quadratic model, and 5) semi-log model.

Given a unique data base such as the Residential Energy Consumption Surveys (RECS), an opportunity is presented to experiment with various functional forms and to determine if and to what extent the various identified energy problems may or may not arise.

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Finally, I dedicate this thesis to Roosevelt V. Robinson. Thank you for your support and friendship. "You give but little when you give of your possessions. It is when you give of yourself that you truly give" (Khalil Gibran).

## CHAPTER I

### INTRODUCTION

The price of energy in the United States historically has been lower than in most other advanced industrial nations. For years abundant supplies contributed to this lower cost. Low prices for energy and a sustained rate of economic growth encouraged the consumption of energy. Due to low prices and ample supplies, Americans were not prone to conserve energy. Durable and capital goods were designed with infinitesimal concern as to the efficient use of energy.

The subject energy is an old phenomenon with a new concern, due to the energy crisis of the '70s. After the second world war, oil output increased rapidly and was marketed at relatively low prices throughout Europe and Asia. Low marketing costs caused rival new firms to enter the market stimulating competitive pricing; technological progress was also reducing costs.

During the '50s and '60s international oil prices declined rapidly. Leaders of the exporting countries became concerned about the decline in prices which caused their oil profits to fall, that they formed an exporter cartel to attempt to do what the companies could not accomplish on their own. By 1960 the cartel had become unsuccessful, but did cut into producers' profits. By the '70s cartel action became increasingly effective in raising the scarcity value of oil, and its price, by taking



actions which induced companies to curtail production, while the demand for oil continually increased, as shown in Table 1. The demand for oil and the restricted supply became substantially equal, causing price pressures to develop.

By October 1973, the cartel solidarity (OPEC) produced the Arab oil embargo. The OPEC cartel was able to hold, and in fact increased, the price of oil. In 1974 crude oil prices were extremely high relative to production costs. Since every industrial country is dependent upon oil imports, raising their prices in effect raised the price of all substitute energy sources as well throughout the world. Owing to the fickle politics of the Middle East, combined with domestic pricing decision and an expected inflationary economy, there is still concern as to the impact of future occurrences.

#### Problem Statement

The problems that unfolded due to the embargo led to a proliferation of studies on energy. Certainly, given the nature of the crisis, the response by researchers was very welcome, except that the information generated by these studies created its own unique problems.

A review of the literature on energy demand shows that there is considerable range in the price and income elasticities that have so far been estimated (Table 2). The short-run price elasticities range from  $-.01$  to  $-.67$ ; the long-run price elasticities range from  $-.25$  to  $-1.86$ ; the short-run income elasticities from  $.03$  to  $.33$ ; and the long-run income elasticities from  $.09$  to  $1.63$ .

TABLE 1

CURRENT AND REAL U. S. AND WORLD CRUDE OIL PRICES, 1880-1977

Year	Current U. S. Price <sup>a</sup> (\$/bbl) (Average Price)	U. S. Consumer Price Index (1957-1959 = 100)	Real U. S. Price in 1957-1959 (dollars/bbl)	World Price Estimated Actual Trans- actions Price	Real-world Price in 1957-1959 (dollars/bbl)
1880	\$ .94	34	\$2.76		
1890	.77	32	2.41		
1900	1.19	39	4.10		
1905	.62	31	2.00		
1910	.61	33	1.85		
1915	.64	35	1.83		
1920	3.07	70	4.40		
1925	1.68	61	2.75		
1930	1.19	58	2.04		
1933	.67	45	1.49		
1940	1.02	49	2.09		
1945	1.22	63	1.95		
1950	2.51	84	3.01	\$1.71	\$ 2.04
1955	2.77	93	2.97	1.63	1.75
1960	2.88	107	2.70	1.53	1.43
1965	2.86	110	2.50	1.33	1.21
1970	3.18	134	2.37	1.26	.94
1971	3.39	145	2.34	1.66	1.14
1972	3.39	150	2.26	1.84	1.23
1973	3.89	154	2.53	2.91	1.89
1974	6.74	175	3.85	10.77	6.15
1975	7.67	187	4.10	10.72	5.73
1976	8.11	202	4.01	11.51	5.70
1977	8.22	211	3.90	13.12	6.22

<sup>a</sup>Average price received by domestic sellers, which is the weighted average of the prices of old oil and new oil.

SOURCES: American Petroleum Institute, Petroleum Fact and Figures, U. S. Bureau of Mines, U. S. Bureau of Labor Statistics, Petroleum Industry Research Foundation, Vertical Divestiture and OPEC (New York, 1977), p. 9.

TABLE 2

## SUMMARY OF PRICE AND INCOME ELASTICITIES OF RESIDENTIAL AND COMMERCIAL DEMAND FOR FUEL OIL

Research Study	Sample	Price Elasticity <sup>a</sup>		Income Elasticity <sup>a</sup>	
		Short-run	Long-run	Short-run	Long-run
I. Reduced-Form Models					
A. Dynamic consumption models					
- Cohn, Hirst, Jackson (1977)	Pooled: annual, states, 1969-74 (Nos. 1-4 fuel oil)	-0.19	-0.51	0.50	1.33
- Taylor, Blattenberger, Verleger (1977)	Pooled: annual, states, 1967-72 (all distillates and No. 2 separately)	n.s.	n.s.	n.s.	n.s.
- Alt, Bopp, Lady (1976)	Time series: monthly, U. S., 1967-74 (all distillates)	-0.13	-0.27	1.26	1.70
B. Fuel shares models					
- Baughman, Joskow (1975)	Pooled: annual, states, 1968-72 (all fuel oils)	-0.18	-1.12	n.s.	n.s.
- Chern (1976)	Pooled: annual, states, 1971-72 (all distillates)		-1.61		n.s.
- DOE (1978)	Pooled: annual states, 1960-75 (all distillates; residential and commercial separated)	R: -0.7 C: -0.3	-1.50 -0.70	n.s.	n.s.
II. Structural Models					
- Anderson (1974)	Pooled: annual, states, 1960-70 (all distillates)		-1.76		

<sup>a</sup>The estimates given are statistically significant at the 0.05 level. An entry of n.s. indicates not significant. A blank space means no estimate was attempted or reported.

SOURCE: Douglas R. Bohi, Analyzing Demand Behavior: A Study of Energy Elasticities (Baltimore: Johns Hopkins University Press, 1981), p. 130.

Bohi has suggested two principal reasons to explain this wide variation in price and income elasticity estimates:

- 1) The differences in the economic and institutional conditions reflected in the sample (different groups, tastes, lifestyles, stock of energy using capital, etc.). As he points out, "If there are forecast errors, they derive from the changes in the sample period used, that is, the structure of demand change."<sup>1</sup>
- 2) The differences in estimation procedure and the functional form imposed on the data. Bohi points out, "a proper specification of the functional form in energy studies is important because in combination with data, the functional form determines the nature of elasticities that are estimated."<sup>2</sup>

The question then becomes: How does one go about making policy recommendations given this wide variation in estimates? Without some guidelines to make such a selection, energy policy outcomes could be costly in view of the large number of people who are affected. An understanding of the effect of functional form on elasticity forecasts is therefore important in generating the information suitable for policy-making implementation.

Even though the differences due to the institutional conditions reflected in the sample are important, in this study we focus on differences in estimation procedures since these are the least understood and more difficult to identify. Specifically we focus on the proper specification of the functional form in combination with the

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<sup>1</sup>Douglas R. Bohi, Analyzing Demand Behavior: A Study of Energy Elasticities (Baltimore: Johns Hopkins University Press, 1981), p. 3.

<sup>2</sup>Ibid.

data because it helps to determine the nature of elasticities that is estimated.

### Objectives

The primary objective of this study is to experiment with alternative specifications of the functional form of energy demand equations and to assess the effects of these specifications on estimated price and income elasticities. We will experiment with the following forms: a) linear functions; b) log-linear specifications; c) quadratic specifications; d) reciprocal specifications; and e) semi-log specifications. In making this assessment, emphasis will be placed on:

- 1) Size and nature of bias when different measures of the same variables are used in specifications; and
- 2) Size and nature of bias when different specifications of the functional form of demand are used.

These different specifications of the demand model will be subject to estimation using the Residential Energy Consumption Survey Data. Since the same data set is being applied to the alternative specifications, we will be in a position to evaluate the possibilities of specification error in using one of the types of functions considered in this paper.

### The Energy Market Outlook 2000

Americans are demanding more and more goods and services that require the use of energy in their productions. "Ever since the oil embargo of 1973, the effect of rising energy prices upon American

households has been a matter of significant public debate.<sup>3</sup> This is especially true with respect to low income households. Due to the instability in crude oil prices, many researchers have tried to forecast price fluctuation through the year 2000. Both industry and official governmental agencies have developed crude oil scenarios for the United States.

According to Conoco, "the dramatic collapse of crude oil markets in the early 1986 was the result of forces set in motion by the high oil prices of previous years."<sup>4</sup> The challenge that faces the United States policymakers will be to develop policies that will allow consumers to benefit from low oil prices while minimizing the dependency on imported oil. Due to price instability, the task may be difficult to achieve by policymakers.

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<sup>3</sup>p. S Arvind Teotia, David South, Dee Wernette, Elliott Levine, and James A. Throgmorton, The Potential Impacts on Selected Energy Conservation Measures on U. S. Minority Households, Argonne National Laboratory, Energy and Environmental Systems Division, October 1984, p. 1. See, for example, "Energy Policy Project of the Food Foundation," A Time to Choose: America's Energy Future (Cambridge, Massachusetts: Ballinger, 1974); D. K. Newman and D. Day, The American Energy Consumer (Cambridge, Massachusetts: Ballinger, 1975); E. S. Grier, Colder... Darker: The Energy Crisis and Low Income Americans (Washington, D. C.: Government Printing Office, 1977); R. Stobaugh and D. Yergin, (eds.) Energy Future: Report of the Energy Policy Project at the Harvard Business School (New York: Random House, 1979), U. S. Office of Technology Assessment, Residential Energy Conservation, 1979; and National Research Council of the National Academy of Sciences, Energy Use: The Human Dimension (San Francisco: W. H. Freeman, 1984).

<sup>4</sup>Conoco, Inc., Coordinating and Planning Department, World Energy Outlook through 2000, September 1986, p. 2.

Conoco's assessment of the world energy outlook can be classified into three cases: 1) low-case, 2) mid-range, and 3) high-case as shown in Figure 1. Based on Figure 1, the low case price per barrel is \$10-\$12. The arrow in Figure 1 indicates a destabilizing oil market if OPEC increases its oil supply by 2000. "If OPEC does not expand capacity to meet demand at lower prices it will control three-fourths of the world oil supply by 2000, and U. S. oil imports will reach 15 million barrels a day."<sup>5</sup>

The mid-range represents fluctuating prices between \$15-\$20 per barrel through 1990. Instability in oil prices could temporarily send prices above \$20 per barrel or below \$15 per barrel if a combination of market forces and production restraint by OPEC would restore prices to the \$15-\$20 per barrel range.

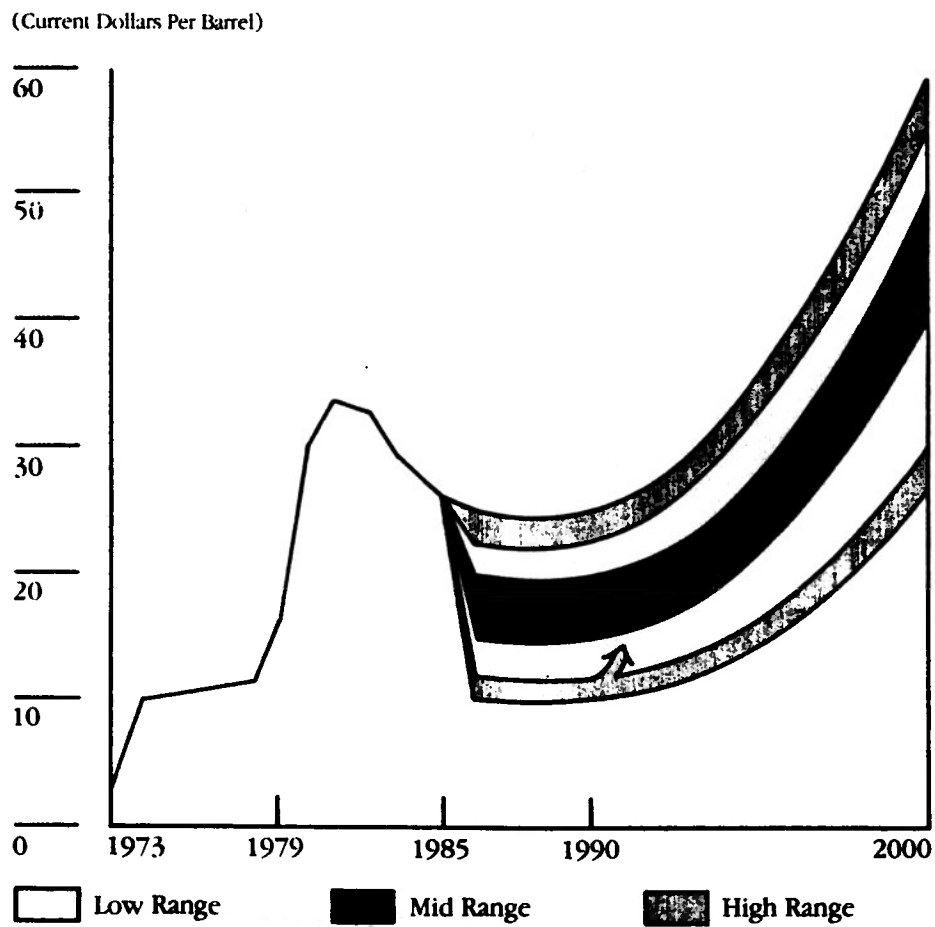
The high-range price will cause crude oil to rise above \$20 per barrel, which indicates that the U. S. would have an overwhelming dependency on OPEC as shown in Figure 2. The Figure states the prospects for world reliance on OPEC and U. S. import dependence under the alternative price scenarios.

Data Resources, Inc., a major energy "think tank," has also developed scenarios for the U. S. energy market. Their model contains more than 2,200 equations. Table 3 indicates the projected crude oil prices used based on the energy model (DRI).

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<sup>5</sup>Ibid., p. 10.

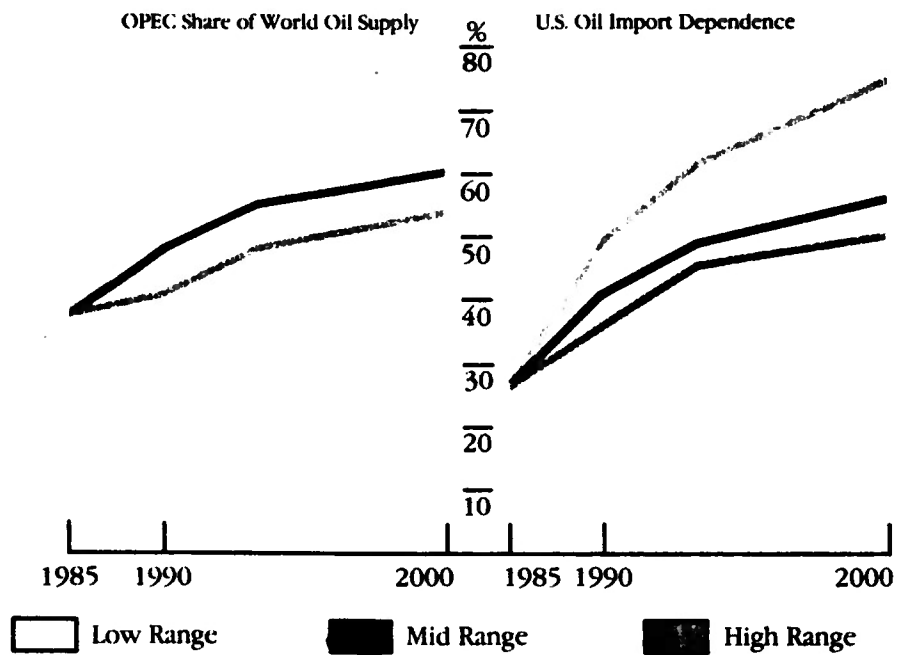
FIGURE 1  
CRUDE OIL PRICE SCENARIOS



SOURCE: Conoco, Inc., Coordinating and Planning Department, World Energy Outlook through 2000, September 1986.



FIGURE 2  
COMPARISON OF ALTERNATIVE SCENARIOS



SOURCE: Conoco, Inc., Coordinating and Planning Department, World Energy Outlook through 2000, September 1986.

TABLE 3  
PROJECTED CRUDE OIL PRICES USED IN ENERGY MODEL SOLUTION

Crude Oil	Average Price (\$/bbl)						Growth Rate <sup>a</sup> (%)
	1981	1982	1983	1984	1985	1990	
<u>1981 \$</u>							
Domestic	34.33	31.21	26.82	25.93	29.55	55.23	7.4
Imported	37.05	33.55	28.60	27.50	31.00	55.95	6.6
Acquisition	32.24	31.87	27.24	26.48	30.03	55.49	7.2
<u>1982 \$</u>							
Imported	39.45	33.70	27.44	25.09	26.76	35.79	0.8

<sup>a</sup>Annual compound rate of growth, 1982-1990.

SOURCE: David Poyer, Residential Fuel Consumption Patterns for Poor, Black and Elderly Households: A Comparative Study vol. 2, Analytical Assessment, Argonne National Laboratory, Energy and Environmental Systems Division, August 1983, p. 100.

As summarized by Poyer,

Prices are expected to decline from \$33.55 barrels in 1982 to \$28.60 in 1983 due to softness in the world oil market. In 1983-84 demand rose slightly from 3.29 million barrels per day in 1982 to 3.99 million barrels per day in 1983, and 4.28 million barrels per day in 1984.<sup>6</sup>

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<sup>6</sup>David Poyer, Residential Fuel Consumption Patterns for Poor, Black and Elderly Households: A Comparative Study vol. 2, Analytical Assessment, Argonne National Laboratory, Energy and Environmental Systems Division, August 1983, p. 99.

In 1985, U. S. oil imports increased to 4.43 million barrels per day. "As a result, OPEC is able to post its first price increase since 1981, pushing the U. S. imported oil price to \$31.00 per barrel in 1985 from a low price of \$27.50 in 1984. By 1990, U. S. crude oil imports is expected to rise to 4.88 million barrels per day and rise to a moderate price increase by OPEC."<sup>7</sup>

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<sup>7</sup>Ibid.

## CHAPTER II

### LITERATURE REVIEW

Since the shock of the Oil Embargo of 1973, researchers have focused on numerous energy-related problems. The literature review will be divided into three parts:

- 1) A general overview of energy demand;
- 2) The theoretical and empirical studies on functional forms; and
- 3) The relevance of the theoretical issues to empirical studies on energy demand.

#### Energy Demand Studies Overview

Bohi's survey<sup>1</sup> of energy studies and an earlier survey by Taylor<sup>2</sup> are instructive reading for a proper understanding of the methodological and functional form problems encountered in energy studies. A few of these issues are discussed to shed some light on the nature of the problems. Some authors, with less comprehensive surveys, have addressed some of the major methodological issues in energy studies. For example,

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<sup>1</sup>Douglas Bohi, Analyzing Demand Behavior: A Study of Energy Elasticities (Baltimore: Johns Hopkins University Press, 1981).

<sup>2</sup>L. D. Taylor, "The Demand for Energy: A Survey of Price and Income Elasticities," ed. William D. Nordhaus in International Studies of the Demand for Energy (Amsterdam: North-Holland, 1977).

since the work of Houthakker,<sup>3</sup> the controversy over whether average or marginal prices is the appropriate independent variable to be used in energy studies is largely unresolved.<sup>4</sup>

Taylor summed up the problem as follows:

The major shortcoming in econometric literature on residential demand for electricity is the failure to deal adequately with decreasing block pricing--the use of single quantity for price electricity--whether an average price or a marginal rate--is not adequate.<sup>5</sup>

As Halvoren demonstrates,

If average and marginal prices are positively correlated (as is the usual case), then the use of one of the prices in absence of the other will lead, in general, to an upward bias in the estimate of the price elasticity--a problem that can be explained using the theorem of omitted variables, the procedure used by Kerry Smith. One solution is to include both the average and marginal prices, but these prices should be taken from actual tariff schedules.<sup>6</sup>

A related price specification issue is related to the use of ex post facto prices (total expenditures/quantity consumed). Here the problem is the resulting simultaneity and identification problems since, in this case, the average price curve and the demand curve both slope

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<sup>3</sup>H. S. Houthakker, P. K. Verleger, D. P. Sheehan, "Dynamic Demand Analysis for Gasoline and Residential Electricity," American Journal of Agricultural Economics 56 (May 1974):412-418.

<sup>4</sup>Robert Halvoren, "Residential Demand for Electric Energy," Review of Economics and Statistics vol. 57, no. 1 (February 1975):9-18.

<sup>5</sup>Lester D. Taylor, "The Demand for Electricity: A Survey," Bell Journal of Economics and Management Sciences vol. 6, no. 1 (Spring 1975):74.

<sup>6</sup>Halvoren, pp. 9-18.

in the same direction. Poyer,<sup>7</sup> arguing in the Halvoresen tradition, has suggested that simultaneous equation specification of the energy demand equation be used to resolve the problem. Green<sup>8</sup> argues that since households tend to consume within the same price block, the simultaneity problems are largely eliminated since the demand curve is identified. In this area also, there is no consensus as to which specification to adopt in studies of energy demand.

Discussion of other issues in the specification of energy demand equations are rather fragmented and have not received the level of attention devoted to the average price versus marginal price issues or the ex post facto pricing issues. Garbacz<sup>9</sup> has pointed out that including both heating degree days and cooling degree days in a demand model using household data can lead to multicollinearity problems. The main effect was on the signs of the coefficients and the size of the standard errors. He included only cooling degree days since few households heat with electricity, while electricity is the overwhelming choice for air conditioning.

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<sup>7</sup>Poyer, p. 14.

<sup>8</sup>Rodney D. Green, Arlease G. Salley, R. Gail Grass and Anthony Osei, "The Price and Income Elasticities of Demand for Home Heating Fuels: A Disaggregated Model Approach," vol. II, Report by the Department of Economics, Howard University, Washington, D. C., September 1984, Appendices.

<sup>9</sup>Christopher Garbacz, "The Model of Residential Demand for Electricity Using a National Household Sample," Energy Economics 5 (April 1983):124-128.

Donnelly points out,

When a common base temperature is used for both heating and cooling degree days, one is forced to record HDD's during middle of summer and CDD's in the winter. The usefulness of the degree day definition is limited since it is based on a simple average of two extreme daily temperatures.<sup>10</sup>

The author, therefore, used his own bases of 12 C and 23 C for measuring HDD and CDD.

### Specification of Functional Forms (Theoretical)

The choice of functional form is important because it determines the nature of elasticities that are estimated.<sup>11</sup> The size of elasticities used in policy analysis depends on informational form so that the choice of functional form becomes critical in assessing the potential impacts of alternative future energy price and quantity restrictions. Discussion of the appropriate functional form is complicated by the fact that the choice is contingent on an absence of well-known problems in econometrics. Functional form also has an impact because of some of the well-known problems which exist in energy studies; they include: 1) aggregation problems; 2) specification error; 3) identification problems; 4) omitted variables; 5) simultaneity problems; 6) multicollinearity; and 7) heteroscedasticity. The

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<sup>10</sup>W. A. Donnelly, "Residential Demand for Electricity: A Variant Parameter Approach," Applied Economics vol. 17, no. 2 (1985):241-242.

<sup>11</sup>Bohi, p. 45.

occurrence of these problems in energy demand studies will be elaborated upon in subsequent paragraphs.

The theoretical discussion on selecting a functional form is extensive and firmly established. Most of the more recent literature focused on the application of the theoretical principles to specific subject matter areas in an effort to lay some guidelines upon which future research could proceed. Two general applications of some of the issues in selecting a functional form are exemplified by the work of Ramsey and Zarembka<sup>12</sup> and Heckman and Polachek.<sup>13</sup> These two studies are briefly reviewed and after that attention is then shifted to the specific area of energy demand.

Ramsey and Zarembka applied various specification error tests to a number of alternative production function models using aggregate U. S. manufacturing data by states for the year 1957. The authors proposed five different production functions in their study: 1) Cobb-Douglas (CD); 2) constant elasticity of substitution (CES); 3) variable elasticity of substitution (VES); 4) generalized production function; and 5) a quadratic production function.

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<sup>12</sup>James B. Ramsey and Paul Zarembka, "Specification Error Tests and Alternative Functional Forms of the Aggregate Production Function," Journal of the American Statistical Association vol. 66, no. 335 (September 1971):473.

<sup>13</sup>J. Heckman and S. Polachek, "Empirical Evidence of Functional Form of Earnings - Schooling Relationships," Journal of American Statistical Society 69 (1974):350-354.



In order to select among the various functional forms, the authors used four well-known Ramsey tests for specification errors. These tests are RESET, RASET, BAMSET, and KOMSET.<sup>14</sup> Since the Ramsey's paper has been exhaustively reviewed in the Appendix, a further discussion is omitted. A fifth test, "chi-square" goodness-of-fit test for normality, was also used.

The authors explain,

The five tests are to a considerable extent complementary since each test, though defined with respect to the same null hypothesis, is a test against a different alternative hypothesis. Basically, RASET and RESET are tests against the alternative hypothesis of a non-null mean vector of the residuals. RESET is a parametric test which requires the assumption that the residuals are normally distributed, whereas RASET is a nonparametric test which does not require such an assumption. KOMSET is a nonparametric test against the same alternative and that a non-normality of the error terms insofar as ratios of the squared residuals will not be distributed as F. BAMSET is a test for heteroscedasticity only. The goodness-of-fit test is one between normal and non-normal density functions for the distributors of the residuals.<sup>15</sup>

To carry out the tests, the authors first discussed what constitutes "full ideal conditions." Consider the basic statistical model of the form:

$$Y = X + U$$

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<sup>14</sup>Ramsey and Zarembka, p. 471. Included in the appendix is the mathematical specification of the Ramsey tests (RESET, RASET, BAMSET, KOMSET and Chi-square goodness-of-fit test)

<sup>15</sup>Ibid.

where,

$Y$  = An  $N \times 1$  vector of observations on the "dependent" variable;

$X$  = An  $N \times K$  matrix of regressors; and

$U$  = The  $N \times 1$  vector of error terms.

Then the "full ideal conditions" are that: 1) the matrix  $X$  is of rank  $K$ ; and 2) the multivariate distribution of the vector  $U$  conditional on the observed regressor matrix is  $N(0, \sigma^2 I_n)$ .

The null hypothesis for each of the five models is that the full ideal conditions are satisfied. Thus, if the  $j$ th model is the model that satisfies the "full ideal conditions," the remaining four models are misspecified as to the functional form and/or normality of the disturbance term.<sup>16</sup> In comparing the five models, the following assumptions are made:

- 1) The  $N$  observations are statistically independent;
- 2) No variables have been omitted;
- 3) The errors in variable problem are statistically insignificant; and
- 4) The simultaneous equation problem is statistically insignificant.

If the first assumption does not hold, then the distributional properties of the test statistic are unknown.<sup>17</sup> Assumptions two and four ensure that the only misspecifications involved in comparing the

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<sup>16</sup>Ibid., p. 473.

<sup>17</sup>Ibid.

five models are those of functional form, heteroscedasticity, and normality of the disturbance term. Assumptions three and four recognize that regressors are observed with error and that the production is merely one function in a simultaneous equation system composed of the production function and the profit maximization conditions. This assumption is often violated in many single equation regressions and the crucial point becomes the seriousness of the error.

The five models were estimated using various statistical techniques and the BLUS (Best, linear, unbiased scalar covariance matrix) disturbance vector was calculated. All the tests were applied at the 10 percent significance level to the residuals so derived.

The study has important conclusions that are applicable to the problems in energy research. These conclusions are distilled purposively:

- 1) The traditional method of using  $R^2$  measures are inadequate. All the models had high  $R^2$  implying "good fit," and yet, based on the tests, there were considerable misspecification.
- 2) None of the models was rejected on the basis of non-normality.
- 3) The conclusions were used to analyze and to discuss firm theory, i.e., economies of scales, returns to scale, the limit of the CES, VES, etc. to the CD.

#### Application to Energy Demand Studies

Different authors have taken different viewpoints on the solution to resolving energy demand problems. The most often discussed energy-related problems include: omitted variable, multicollinearity, identification problems, and aggregation problems.

### Omitted Variables

The omitted variable problem is very important in energy studies. The problem occurs largely due to that cited by Theil, that is, a "researcher may not have observations of the variables,"<sup>18</sup> and therefore decides to omit it. For brevity this discussion will focus on: 1) Reasons why a price variable may be omitted, and 2) reasons why a quantity variable may be omitted.

Since the work of Houthakker,<sup>19</sup> the conventional view is that the "marginal price" and not "average price" should be used in energy studies. The theoretical arguments offered to justify this conclusion are extensive.<sup>20</sup> The crux of the argument is that,

A single marginal price is relevant to a consumer's decision only when he is consuming in the block to which it attaches; it governs behavior while the consumer is in that block, but it does not, in and of itself, determine why he consumes in that block as opposed to some other block.<sup>21</sup>

Most studies use average prices usually constructed by dividing quantity of energy consumed into total expenditure. When this procedure is used, the marginal prices are omitted even though there is a

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<sup>18</sup>H. Theil, Principles of Econometrics (New York: John Wiley, 1971), p. 1.

<sup>19</sup>Houthakker, Verleger, Sheehan, pp. 412-418.

<sup>20</sup>Bohi, Analyzing Demand Behavior, and Taylor, "The Demand for Energy," p. 3.

<sup>21</sup>L. D. Taylor, "The Demand for Electricity: A Survey," The Bell Journal of Economics and Management Sciences vol. 6, no. 1 (Spring 1975):9, 74

suggestion that, "a simple but yet substantially correct procedure is to include both a marginal and an average price as predictors in the demand function."<sup>22</sup> If a researcher uses average or marginal price alone, and if these two prices are positively correlative (as is likely to be the case) then, in general, there will be an upward bias in the estimate of the price elasticity. "This outcome is based on the discussion of the omitted variables so, in effect, the problem of which price variable to use in energy demand equations reduces to a problem of the impact of an omitted variable."<sup>23</sup>

Another way an omitted variable problem can arise is through the use of proxies in energy studies. The specification of what "quantity" of energy one is referring to is not exactly clear. However, this aspect of the problem has not received the degree of attention needed. If one is using the Residential Energy Consumption Survey (RECS), this issue will not arise because the quantities are specified in the survey.

This is not true with other data sources like the Annual Housing Survey (AHS), since this data base includes only annual or monthly expenditures on each fuel. Green<sup>24</sup> has explained in detail how bias arises as a result of using constructed quantity variables as proxies for actual quantities consumed. The author points out,

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<sup>22</sup>Taylor, "The Demand for Electricity," p. 79.

<sup>23</sup>Ibid., p. 80.

<sup>24</sup>Green, Salley, Grass and Osei, p. 80.

The difference between the proxy for quantity and the true quantity follows the pattern that for high levels of expenditures (when marginal price falls), the constructed quantity will understate the true quantity. Similarly, at low expenditure levels, the constructed quantity may overstate the true quantity.<sup>25</sup>

The net effect is that estimates of elasticity will be biased downwards. This result follows on the analogy of the discussion on price (i.e., if the construction quantity variable is correlated with the true quantity, the results of the elasticity will be biased downwards).

### Multicollinearity

Multicollinearity problems are relevant to specification problems for several reasons. Their presence or suspicion of their presence may lead a researcher to omit a relevant variable, thereby committing an omitted variable's problem. If, however, the problem exists so that the regressor matrix is not full rank, the test is not invalid but there is considerable complexity introduced into the test procedure.<sup>26</sup>

In the energy literature, Garbacz is one of the few authors to discuss the subject of multicollinearity in the specification of the energy demand model.<sup>27</sup> The author found that including the weather

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<sup>25</sup>Ibid.

<sup>26</sup>Ramsey and Zarembka, p. 471.

<sup>27</sup>Christopher Garbacz, "In Search of Residential Electricity Demand," presented to North American Meeting of the International Association of Energy Economists, Denver, Colorado, November 18-19, 1982.

variable heating degree days (HDD) and cooling degree days (CDD) in a national household data set led to collinearity problem. Figure 3 shows the U. S. Weather Zone Map of Heating Degree Days and Cooling Degree Days of the United States. Including both variables led to unexpected signs on the coefficients and very large standard errors. In this study, only CDD was used. Garbacz also suggests that including both variables in state-based models may be correct. The author did not include size of dwelling since this created distortion in the estimated coefficients of income due to probable multicollinearity problems.

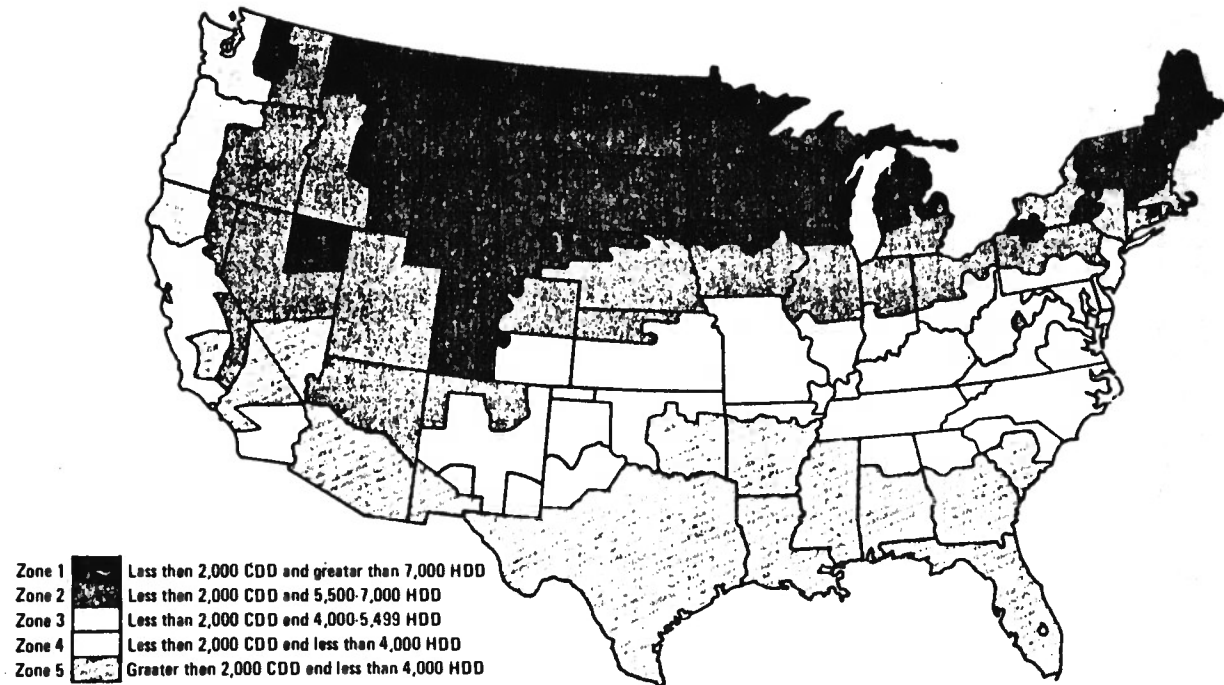
#### Identification Problems

This is a more exciting problem because it is more amenable to empirical analysis. This problem comes up most often in studies on electricity and natural gas since in both markets block-rate pricing is used. In its barest essentials, what happens is that for these two energy goods, because of the block-rate pricing mechanism, large quantities are consumed at the lower block rates so that one ends up with prices inversely related to quantities on both supply and demand side, and it is impossible under these circumstances to determine whether a supply or demand curve has been estimated.

The implication of the block-rate pricing problem is that in studies on electricity and natural gas, scholars have not yet agreed as to whether the average, marginal or total price is to be used in measuring the price factors. Scholars have suggested that if the

FIGURE 3

U. S. WEATHER ZONE MAP OF HEATING DEGREE DAYS (HDD) AND COOLING DEGREE DAYS (CDD)



Note: Heating degree days (HDD) refers to the number of degrees the daily average temperature is below 65 degrees Fahrenheit. Cooling degree days (CDD) refers to the number of degrees the daily average temperature is above 65 degrees Fahrenheit.

SOURCE: Chart prepared by U. S. Bureau of the Census



specification of the price variables is not used, there will be bias in the estimated parameters and hence in forecasts.

Poyer, following several other scholars, used average price relying on the fact that since "his demand equation is a constant elasticity of substitution, the estimated elasticities using average price data are identical to those using marginal price data."<sup>28</sup> Poyer imposed a three-stage least squares procedure in his estimation to obtain efficient estimates of the demand parameters.

Green, on the other hand, used more traditional arguments to eliminate the supply-side effects and hence justify the reduced form estimation procedure. Green points out, the reduced form, single equation linear model used to reflect demand for gas and electricity is justified because the rate structures of the fuel companies are regulated by public utility commission; price thus cannot be considered an endogeneous variable as in a competitive market. In addition, supply can be said to be perfect elastic because of the excess capacity of utilities; thus, the interdependency of price and quantity is eliminated. With excess capacity, the level of quantity demanded does not affect the price. Thus, we can use a simple-form demand model to reflect the market rather than a multi-equation system.<sup>29</sup>

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<sup>28</sup>Poyer, p. 93.

<sup>29</sup>Green, Salley, Grass and Osei, p. 40.

Taylor, who examined these issues in greater detail than both Poyer and Green, suggests that,

The marginal price is relevant to a consumer's decision only when he is consuming in the block which it attaches; it governs behavior while consumer is in the block, but it does not, in and of itself, determine why he consumes in the block as opposed to some other block.<sup>30</sup>

The issue as Taylor sees it is empirical. The following observations by the author are especially relevant to our present effort.

- 1) That the bias introduced, as a result of using average or marginal prices, is similar to an omitted variable's situation when the appropriate specification requires more than a single measure such as marginal or an average price. Specifically, the author notes if:
  - A) The omitted component is positively related to price, there will be a positive covariance between the price variable and the error term. The result will be a negative bias added to the expected value of the estimated price coefficient and the estimate will be too large;
  - B) If omitted component is inversely correlated with price, a positive bias would result; and
  - C) Since the appropriate specification is unknown, and because the included price may be positively or negatively correlated with omitted variables, one cannot predict whether bias will be positive or negative. In effect, the determination of the nature of bias depends on the absence of any other specification errors in the equation.

Thus, if the problems exist in the equation: 1) aggregation error; 2) inappropriate functional form; 3) other omitted variables; 4) measurement error; and 5) disturbance term, that is, not independently and identically disturbed; then the effect of an omitted price term becomes

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<sup>30</sup>Taylor, "The Demand for Energy," p. 74.

uncertain. What the summarized review of the opinions of scholars is suggesting is that the issue of determining what prices to use or how to measure some variables in energy studies, contains too many degrees of freedom and some explanation is wanting. The disagreements serve only as source of inspiration for our current effort.

### Aggregation Problems

Aggregation problems generally arise when information is summed over individuals or goods in statistical analysis. Specifically, it is not necessarily true that,

- 1) The price and income elasticities within micro relations will carry over as an aggregate price elasticity in the aggregate relations; and
- 2) There exist an aggregation bias in regression estimate so that the expected value of the aggregation parameter estimates is equal to the sum of the micro estimates plus some covariance term involving the micro data.

Several methods for dealing with aggregation problems have been suggested. These depend upon whether we are concerned with aggregation over individuals or goods. If aggregation is over commodities, the composite commodity theorem<sup>31</sup> suggests that the relative price of individual commodities in the group remains constant. Gorman<sup>32</sup> and

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<sup>31</sup>J. R. Hicks, Value and Capital (Oxford: Oxford University Press, 1939), reprinted 1974, p. 1.

<sup>32</sup>W. M. Gorman, "Separable Utility and Aggregation," Econometrica vol. 27, no. 3 (July 1959):469-481.

Strotz<sup>33</sup> suggest that the individual utility functions be strongly separable so that changes in prices and income will not affect the proportion of expenditures on each good within each group. This is a very restrictive assumption because it implies that if two goods belong to different groups, the marginal utility associated with each independent variable of the used price and quantity indices based on group value shares and marginal utility budget shares, average budget shares are weights. Since little is known about marginal budget shares, average budget shares are used. In a demand function it is necessary that all consumers within the same group have identical income elasticities. If this holds, the aggregate demand function will depend on the individual demand functions plus the distribution of consumers over price, income, and quantities consumed of other goods. Barten<sup>34</sup> uses a less restricted assumption, weak separability to address the aggregation issue. According to this approach, aggregation is permitted if for any two goods belonging to a community group, the ratio of marginal utilities is independent of the quantity consumed of any good outside the group. In Barten's study, average and marginal budget shares are equal only if utility is strongly separable.

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<sup>33</sup>Robert H. Strotz, "The Utility Tree - A Correction and Further Appraisal," Econometrica vol. 27, no. 3 (July 1959):482-488.

<sup>34</sup>A. P. Barten, "Reflexions sur le construction d'un systeme empirique des fonctions de demande," Cahiers du Seminaire d'Econometrice no. 12 (1970).

Gorman<sup>35</sup> focuses on aggregate over individuals. According to the author, in order to construct an aggregate demand function, it is necessary that consumers within the same group have identical income elasticities.

Aggregation problems are difficult to deal with, specifically because it is directly anchored in some precise knowledge about consumer preferences and attitudes to goods consumption. As a result, the major statistical efforts in energy demand studies have been focused on specification errors and sampling errors.

Most authors of energy papers recognize this problem and usually dismiss it with caution about interpreting their estimated coefficients, others rationalize the problem by using information about the supply-side of energy market. Some of the considerations in eliminating the supply-side are:

- 1) Since the energy industries are public utilities, prices are regulated so the supply-side prices are not relevant;
- 2) Based on consumption history, marginal prices do not count since consumption is largely determined by previous prices;
- 3) Since supply lags take a long time, a perfectly elastic supply curve can be imposed in the short-run, thus the short-run demand curve will be identified.

All the above arguments hold in the short-run only. This identification issue is of less concern in the present study.

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<sup>35</sup>Gorman, pp. 469-481.

Summary

The first part of the literature review deals with the overview of general issues of energy demand. In the second part of the literature review, the theoretical literature on specification error test was reviewed. Based on Ramsey and Zarembka and Taylor, it was concluded that analysis of functional form problems are meaningful only if other specification problems are addressed. The most often discussed specification problems include omitted variables, multicollinearity, aggregation problems, and identification problems. Specific tests developed to identify these problems in empirical research include RASET, RESET, BASET, KOMSET, and the Chi-square good-of-fitness test.

The third part of the review of literature revealed that some of the theoretical issues raised in the second part actually plague empirical studies on energy. The views, findings and suggestions by various researchers have been concatenated to reflect the theoretical problems raised in the second part.

Given a unique data base such as the Residential Energy Consumption Surveys (RECS), an opportunity is presented to experiment with various functional forms and to determine whether and to what extent the various identified energy problems may or may not arise.

### CHAPTER III

#### THEORETICAL FRAMEWORK

The theory of consumer behavior and demand is built on the principal assumption that a consumer attempt to allocate limited money income among available goods and services so as to maximize satisfaction. The problem of resource allocation has prompted economists to stress the price-quantity demand relationship because of the fundamental role in policymaking decisions.

The problem faced by consumers is that of choosing the level of consumption of goods and services so as to maximize utility subject to a given budget constraint. In other words, the consumer arranges his/her purchases to maximize satisfaction subject to his/her budget constraint. For example, the case of two goods-- $X_1$  and  $X_2$ --purchased by consumers can then be stated as:

$$\text{Max } U (X_1, X_2) \text{ subject to } P_1 X_1 + P_2 X_2 = Y$$

where,

$U (X_1, X_2)$  = Utility function;

$P_1, P_2$  = Prices of two goods; and

$Y$  = Level of income.

The utility function is an ordinal representation of tastes in that consumers prefer a bundle of goods  $X (X_1, X_2)$  with a high value

of utility to a bundle with a lower value of utility. The budget constraint requires total expenditures to equal income.

A geometrical explanation of this ordinal representation is illustrated in Figure 4.

#### Economic Model and Estimation

Economic theory predicts that the quantity of energy consumed is a function of the price of energy, the income of the household and other factors. The other factors that have been hypothesized to influence energy demand include weather factors, geographical factors, family size, stock appliances, prices of other fuels, the integrity of the home, etc. All of these factors and many more are potential candidates for entry into the demand function.

Before an examination of specification problems in energy demand is introduced, the economic model is stated as follows:

$$Q_e = f(P_e, \text{Inc}, \text{CDD}, \text{HDD})$$

where,

$Q_e$  = Quantity of energy demand by household;

$P_e$  = Price of energy;

Inc = Level of income;

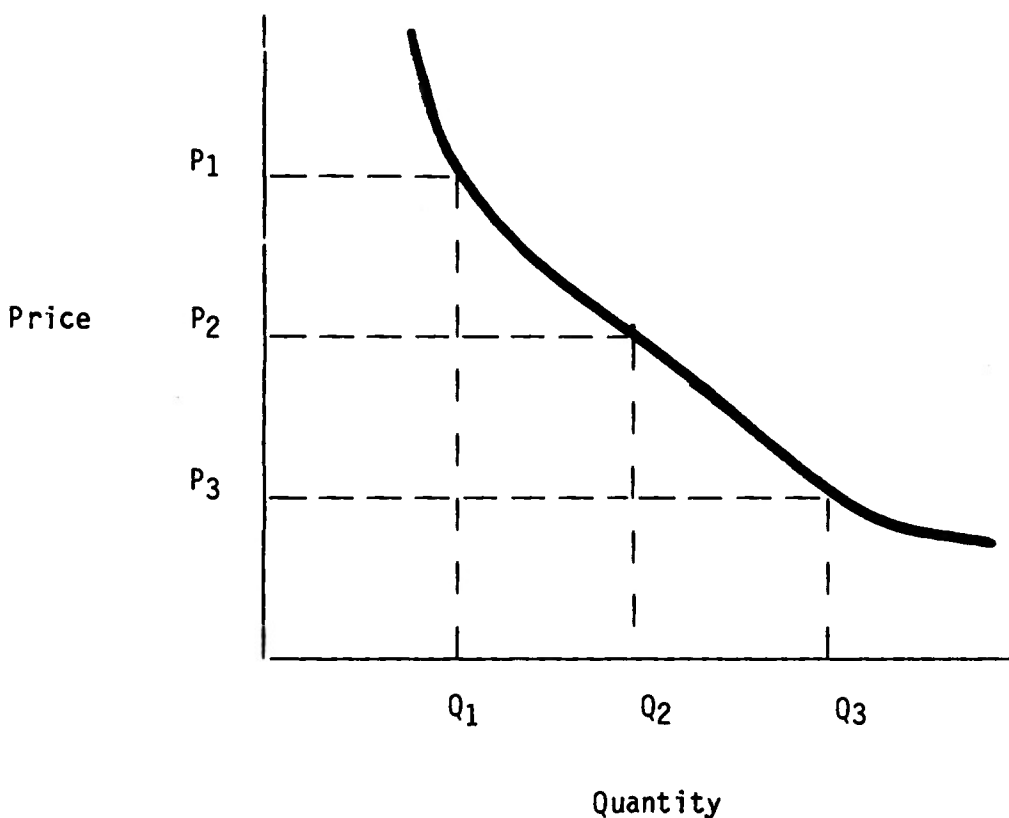
CDD = Cooling degree days; and

HDD = Heating degree days.

The weather variables have been included because energy use in the home is largely determined by weather variations.



FIGURE 4  
THE DEMAND CURVE<sup>1</sup>



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<sup>1</sup>C. E. Ferguson and J. P. Gould, Microeconomics Theory, 4th ed. (Homewood, Illinois: Richard D. Irwin, Inc., 1975), p. 45. According to Ferguson and Gould, the demand curve for a specific commodity relates equilibrium quantities bought to the market price of the commodity, nominal money income and the nominal prices of other commodities held constant.

### Heating Degree Days

Heating degree days (HDD) is the amount of heat necessary to raise the temperature to 65 degrees (°) Fahrenheit (F) which is the established average base temperature for the day. This means that the temperature of the day is obtained and then subtracted by 65. The difference is the amount of HDD for that day, i.e., if the high was 80° and the low 20°, the average is 50°. As such there are 15 HDD. The higher the HDD, the greater is the amount of heat required to heat the house or space.

### Cooling Degree Days

Cooling degree days (CDD) is the summer analog of the HDD. It serves as an index of air conditioning requirements during the year's warm months. In this case, the mean temperature is obtained and base 65 is subtracted from it. The difference is the CDD. The higher the CDD, the greater will be the energy requirement to reduce the indoor temperature to a comfortable level.

### Statistical Estimation (Specification and Functional Forms)

The differences between statistical estimates of price and income elasticities arise from two basic sources:

- 1) The differences in the economics and institutional conditions reflected in the sample (specification problems); and

- 2) The differences in the procedure applied to the data to derive the estimates (estimation problems).<sup>2</sup>

The differences in estimation procedures are least understood and more difficult to identify. Every econometric model of energy derived starts with the same basic concept. This concept is the differences in estimation procedures that produce divergence.

There is a growing consensus that the double-log specification of the energy demand equation is the correct specification. This specification was used by Poyer in his study on minority energy demand. The price and income elasticities estimated from Poyer's specification will be the base measures against which price and income elasticities from specification will be compared. The mathematical specification of the double-log function is:

$$\text{Log } Q = a + b \log P + c \log Y + d \log Z$$

where,

Q = Quantity demanded;

P = Price;

Y = Income;

Z = Vector of other variables; and

a, b, c and d = Parameters to be estimated.

One disadvantage of the double-log function is the assumption of constant price and income elasticities implied by the model. A variable

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<sup>2</sup>Bohi, p. 3.

elasticity version (VES) of the above specification has been suggested by Mount, Chapman and Tyrrell.<sup>3</sup> A mathematical specification of the VES is as follows:

$$\text{Log } Q = a + b \log P + c \log Y + d \log Z + e 1/P + f 1/Y + g 1/Z$$

where all variables are as previously defined and the specification includes the inverse of price, income, and other variables as additional explanatory variables.

In the previous study by Houthakker,<sup>4</sup> the author considered specification of an inverse model, while Donnelly's is a linear model. A mathematical specification of the inverse model is as follows:

$$Q = aY + b/P + cg + dh + t$$

Q = Quantity of energy consumed by a household;

Y = Income;

P = Price;

g = Capital stock variables;

h = Other variables; and

t = Disturbance term.

Household demand for energy is assumed to be a linear function of own price, income, and in the case of space heating and air conditioning,

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<sup>3</sup>T. D. Mount, L. D. Chapman and T. J. Tyrrell, Electricity Demand in the United States: An Econometric Analysis, Report ORNL-NSF-EP-49 (Oak Ridge, Tennessee: Oak Ridge National Laboratory, 1973).

<sup>4</sup>H. S. Houthakker, "Electricity Tariffs in Theory and Practice," The Economic Journal vol. 61, no. 241 (March 1951):1-25.

of heating and cooling degree-days. A mathematical specification of the linear model can be expressed by the work of Donnelly:<sup>5</sup>

$$Q_e = f (P_e, PC_s, PS_s, Y, X)$$

where,

$Q_e$  = Quantity of electricity consumed;

$P_e$  = Price of electricity;

$PC$  = Price of complementary goods;

$PS$  = Price of substitute goods;

$Y$  = Income; and

$X$  = Other variables (climate, stock of electric appliances).

The theoretical issue with price centers on the fact that the consumer's quantity decision is made simultaneously with unit price determination.

The two climatic factors based on degree-day concept was used. When a common base temperature is used for both heating and cooling degree days as done in the United States, the usefulness of the degree day definition is limited, according to Donnelly, since it is based on a simple average of two extreme daily temperatures. Therefore, the author used her own bases of 12° C for HDD and 23° C for CDD. This was estimated in several linear and log-linear forms.

There is also a suggestion that semi-log function (linear-log) be used in residential demand analysis. A mathematical specification is

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<sup>5</sup>Donnelly, pp. 241-242.

of the form:

$$Y = B_0 + B_1 \ln X$$

According to many authors this functional form is somewhat difficult to interpret. It states that a given percentage change in X results in the same absolute change in Y.

Finally, another form of equation that has been employed in energy studies is the quadratic equation. A mathematical specification is of the form:

$$Q = a + b P + c + p^2 + Y$$

In the specification, two empirical variables are required to measure the one generic variable P. There are three major reasons for using the quadratic specification:<sup>6</sup>

- 1) To capture turning points;
- 2) To allow for a non-linear relation between the independent and dependent variables; and
- 3) To approximate a linear relationship.

### Hypotheses

Hypothesis testing is an essential tool for testing economic data. According to Johnson, Johnson and Buse,

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<sup>6</sup>Aaron C. Johnson, Jr., Marvin B. Johnson and Rueben C. Buse, Econometrics: Basic and Applied (New York: Macmillan Publishing Company, 1987), p. 243.

The essence of testing hypothesis is determining whether an estimate is close enough to the value of the parameter specified in the null hypothesis for it to be reasonable to conclude that the sample was drawn from a population where the null hypothesis is true.<sup>7</sup>

The primary hypotheses based on the economic models described are:

- 1) Price is negatively related to demand for energy. (There is an inverse relationship between price and quantity demanded.)
- 2) Income elasticities should be positive and inelastic.
- 3) HDD variable should be positive and inelastic.
- 4) CDD variable should be positive and inelastic.
- 5) Alternative functional forms will provide different income and price elasticities for energy demand.

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<sup>7</sup>Ibid., p. 131.

## CHAPTER IV

### RESULTS

#### Data

The data used to estimate the elasticities and marginal effects of each functional form used in this paper (double-log, VES, linear, semi-log, reciprocal and quadratic) were provided by the United States Department of Energy. The conception of the Residential Energy Consumption Survey (RECS) in 1980-81 came as an exigent of the energy crisis and price increases of the 1970s. The data provided information to assist federal, state and local municipalities on the policies regarding energy.

The United States Energy Information Administration (EIA) conducted the survey. Some 7,232 households composed the original sample, and 6,051 were officially interviewed. The survey response rate was 92 percent.

The survey is a multi-stage probability sample with two stages to the sampling procedure: 1) Primary Sampling Units (PSUs) and 2) Minor Civil Division (MCDs). The first stage PSUs were large in population because they comprised all or parts of metropolitan areas. The second stage MCDs included cities, townships and towns, and other Census divisions were selected within each PSU. The final sample contained



some 6,051 households interviewed representing an estimated 81,645,000 households in the United States at the time of the survey.<sup>1</sup>

The Residential Energy Consumption Survey, the 1980 Census, along with the Annual Housing Survey, provide the best available national data concerning residential energy consumption and expenditures.

### Results

The linear equation estimated results are:

$$Q = 75.98 - 9.97 P + 0.0016 Y + 0.19 CDD + 0.13 HDD^*$$

(32.07) (-76.19) (50.61) (25.29) (45.89)

(\*Numbers in parentheses are t-statistics.)

The marginal effect which expresses the slope or derivative of the linear equation is a constant. The marginal effect ( $\partial Q / \partial P$ ) of this equation is -9.97. The sign on the price coefficient follows the stated hypothesis and economic theory. As shown in the equation, price coefficient is negative, and the quantity changes in the opposite direction. There is an inverse relationship between price and quantity.

Price elasticity of demand is useful measure of the responsiveness of the quantity demanded of a particular commodity to its price. In

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<sup>1</sup>p. S. Arvind Teotia, David South, Dee Wernette, Elliott Levine and James A. Throgmorton, The Potential Impacts on Selected Energy Conservation Measures on U. S. Minority Households, Argonne National Laboratory, Energy and Environmental Systems Division, October 1984, p. 64.

economics, the meaning has always been stated as the ratio of relative change in an independent variable. In other words, own price elasticity of demand can be defined as:

$$E = \frac{\% \text{ change in } Q}{\% \text{ change in } P} = \frac{\Delta Q}{\Delta P} \cdot \frac{\bar{P}}{\bar{Q}}$$

In the equation, the implied price elasticity of demand is computed as:

$$E = -9.97 \cdot \frac{6.43}{122.5^*} = -.52 \quad \dots\dots\dots (1)$$

(\*These numbers represent the OLS regression on the national data computed for RECS.  $Q = 122.51$ ,  $P = 6.43$  and  $Y = 20400$ .)

Income elasticity of demand can be defined as "the rate of change of quantity with respect to change in income, other determinants remaining constant. The implied income elasticity is computed as:

$$E_y = .0016 \cdot \frac{20400}{122.5} = .27 \quad \dots\dots\dots (2)$$

The weather variables are .19 CDD and 0.013 HDD. Both weather variables are positive and inelastic.

The coefficient of determination  $R^2$  is .38.  $R^2$  is thus the squared correlation (Pearson correlation) between observed and expected values for the dependent variable. It also measures the "goodness" of an estimated regression equation. The coefficient of determination,  $R^2$ , will always be positive.

### The Double-Log Equation

The double-log equation estimated results are:

$$\begin{array}{ccccccc} \ln Q = & 0.074 & - 0.84 & P & + & 0.23 & Y & + & 0.13 & CDD & + & 0.34 & HDD \\ & & (-0.74) & (-98.31) & & (53.96) & (33.74) & & (45.94) \end{array}$$

The double-log specification has a number of appealing features. The double-log specification is most commonly used because it allows for non-linearity and non-additivity in the variables, an economic property of great importance. These important features cannot, as it stands, be estimated by OLS (ordinary least squares) procedures, the key is to take the natural logarithm which will transform the variables into a linear equation. When the variables are transformed, it yields BLUE (BEST LINEAR UNBIAS ESTIMATES) of the coefficients.

As shown in Poyer's literature, the elasticity is a constant elasticity function. Often the constant elasticity property is useful because it is easy to understand, to estimate and to interpret. The model is applicable only if all the values of Q and P are positive, no values can be zero or negative.

The marginal effect of the equation is:

$$\frac{\partial Q}{\partial P} = -0.84 \cdot \left( \frac{122.51}{6.43} \right) = \frac{102.9}{6.43} = -16.0$$

The implied price elasticity is -0.84 and the implied income elasticity is .23. This is the beauty of this functional form because after transformation, the elasticity can be directly interpreted from the results.

They are interpreted as direct elasticities.

The weather variables follow economic theory and  $R^2$  is reported as .46. In this model, the independent variables can explain the dependent variable by .46 percent. All of the signs in the equation are consistent and each value is significant.

The Reciprocal (inverse) results are:

$$Q = -66.37 + 443.31 P + .002 Y + 0.02 CDD + 0.013 HDD$$

$$(-26.28) \quad (72.72) \quad (53.96) \quad (22.75) \quad (43.95)$$

The price coefficient in the equation is positive (443.31 P), stating that the function is decreasing at a decreasing rate. The marginal effect can be stated as thus:

$$\frac{\partial y}{\partial x} = -1 \frac{\partial^2}{\partial x^2}$$

where,

$$\frac{\partial Q}{\partial P} = -443.31 \cdot \frac{1}{(6.43)^2} = -10.72 \quad \dots\dots\dots (3)$$

The implied elasticity of demand is:

$$E = \frac{433.31}{6.49 \cdot 122.51} = \frac{443.31}{795.09} = -0.56 \quad \dots\dots\dots (4)$$

The implied income elasticity is computed as:

$$E_y = .002 \cdot \frac{20400}{122.5} = \frac{40.8}{122.5} = .33 \quad \dots\dots\dots (5)$$

The weather variables are positive and inelastic based on the hypothesis and economic theory. The  $R^2$  reported from the results is .36.

Quadratic

The quadratic equation estimated results are:

$$Q = 6.49 - 15.00 P + 0.002 Y + 0.02 CDD + 0.01 HDD$$
$$(95.91)(-53.03) \quad (51.61) \quad (26.57) \quad (45.93)$$

The quadratic specification represents the functional form:

$$Q = f(P, P^2)$$

where,

$$Q = -15.0 P + 0.25 P^2$$

The marginal effect of the quadratic equation is:

$$\frac{\partial Q}{\partial P} = -15.0 + 2(0.25)(6.43) = -11.75 \dots\dots\dots (6)$$

The implied price elasticity is:

$$E = -15.0 + .50(6.43) \times \frac{6.43}{122.51} = -0.62 \dots\dots\dots (7)$$

The implied income elasticity is:

$$E_y = .002 \times \frac{20400}{122.5} = .33 \dots\dots\dots (8)$$

The weather variable follows economic judgement and the coefficient of determination  $R^2$  is .39.

Semi-log results are stated as:

$$\ln Q = 4.44 - 0.12 P + .000014 Y + 0.0002 CDD + .00011 HDD$$

$$(228.87)(-111.75) \quad (53.13) \quad (30.47) \quad (47.41)$$

The dependent variable is in log form, but the independent variables are not in log form. In this equation, price increases but Q continues to increase, however, at a decreasing rate. The marginal effect of the semi-log equation is:

$$\frac{\partial(\ln Q)}{\partial P} = \left( \frac{1}{Q} \cdot \frac{dQ}{P} \right) P = -0.12 \cdot \frac{1}{6.43} = \frac{-0.12}{6.43} = -.018 \quad \dots (9)$$

The implied price elasticity is:

$$E = -0.12 \cdot \frac{1}{122.5} = \frac{-0.12}{122.5} = -.001 \quad \dots (10)$$

The implied income elasticity is:

$$E_y = .000014 \cdot \frac{20400}{122.5} = \frac{.02856}{122.5} = .00023 \quad \dots (11)$$

The weather variables are positive and inelastic, following the basic theory in economics. The  $R^2$  is .50.

Finally Tables 4, 5, 6 and 7 give a summarized illustration of the results reported in this chapter.

TABLE 4  
ALTERNATIVE SPECIFICATIONS OF THE SAME SIMPLE REGRESSION:  $y = f(x)$

Functional Form	Estimated Equation*
Linear	$Q = 75.98 - 9.97 P + 0.0016 Y + 0.19 CDD + 0.013 HDD$ $(32.07)(-76.19) (50.61) (25.29) (45.89)$
Double-Log (Log-linear)	$\ln Q = -0.074 - 0.84 P + 0.23 Y + 0.18 CDD + 0.34 HDD$ $(-0.74)(-98.31) (53.96) (33.74) (45.94)$
Reciprocal (Inverse)	$Q = -66.37 + 443.31 P + .002 Y + 0.02 CDD + 0.13 HDD$ $(-26.28) (72.72) (51.99) (22.75) (43.94)$
Quadratic	$Q = 6.49 - 15.00 P + 0.002 Y + 0.002 CDD + 0.01 HDD$ $(95.91) (-53.03) (51.61) (26.57) (45.93)$
Semi-log	$\ln Q = 4.44 - 0.12P + .000014 Y + 0.0002 CDD + .00011 HDD$ $(228.87)(111.75)(53.13) (30.47) (47.41)$

\*Numbers in parentheses are t-statistics and

Q = Quantity of energy in BTUs

P = Price of energy per unit

Y = Average per capita income

CDD = Cooling degree days

HDD = Heating degree days

TABLE 5

MARGINAL EFFECT OF PRICE ON QUANTITY FOR EQUATIONS IN TABLE 4

Functional Form	Marginal Effect (Price)
Linear	-9.97
Double-log (Log-linear)	-16.0
Reciprocal (Inverse)	-10.72
Quadratic	-11.75
Semi-log (Linear-log)	-0.12



TABLE 6  
COMPARISON COMMON FUNCTION FORMS

Name	Linear and Additive	Marginal	Elasticity
Linear	$Y = \alpha_0 + \alpha_1 X$ $Q = 75.98 - 9.97 P + 0.0016 Y$	$\alpha_1$ -9.97	$\alpha_1 X/Y$ -0.5233
Double-log	$\ln Y = \ln \alpha_0 + \alpha_1 \ln X$ $= -0.074 - 0.84 (1/P)$	$\alpha_1 Y/X$ -15.86	$\alpha_1$ 0.84
Reciprocal (Inverse)	$Y = \alpha_0 + \alpha_1 / X$ $Q = -66.37 + 443.31 (1/P)$	$-\alpha_1 / X^2$ -10.72	$1/XY$ -0.56
Quadratic	$Y = \alpha_0 + \alpha_1 X + \alpha_2 X^2$ $Q = 95.91 - 15.0 P + 0.25 P^2$	$\alpha_1 + 2 \alpha_2 X$ -11.75	$(\alpha_1 + 2 \alpha_2 X) X/Y$ -0.62
Semi-log (Linear-log)	$Y = \alpha_0 + \alpha_1 \ln X$ $= 4.441 - 0.12 (1/P)$	$\alpha_1 / X$ -0.012	$\alpha_1 / X$ -0.001

TABLE 7

COEFFICIENT ESTIMATES AND IMPLIED PRICE AND INCOME ELASTICITIES FOR  
ALTERNATIVE SPECIFICATION OF THE ENERGY DEMAND MODEL\*

Model	Dependent Variable	Intercept	Price Coefficient (P)	Implied Price Elasticity (E)	Income Coefficient (Y)	Implied Income Elasticity (Ey)	Cooling Degree-Days (CDD)	Heating Degree-Days (HDD)	Coefficient Determination R <sup>2</sup>
Linear	Quantity of Energy	75.98 (32.07)	-9.97 (-76.19)	-.52	0.0016 (50.61)	.27	0.019 (25.29)	0.013 (45.89)	.38
Double-log	Log Quantity of Energy	-0.074 (-0.74)	-0.84 (-98.31)	-0.84	.23 (53.96)	.23	(0.18) (33.74)	0.34 (45.94)	.46
Reciprocal (Inverse)	Quantity of Energy	-66.37 (-26.28)	443.31 (72.72)	-0.56	.002 (51.99)	.33	.02 (22.75)	.013 (43.94)	.36
Quadratic	Quantity of Energy	6.49 (95.91)	-15.00 (53.03)	-0.62	.002 (51.61)	.33	.02 (26.57)	.01 (45.93)	.39
Semi-log	Log Quantity of Energy	4.44 (228.87)	-0.12 (-111.75)	-.001	.000014 (53.13)	.00023	.0002 (30.47)	.00011 (47.41)	.50

\* Numbers in parentheses are t-statistics.

## CHAPTER V

### CONCLUSIONS

The main objective of this study was to experiment with alternative specifications of the functional forms of energy demand equations and assess the effects of these specifications on estimated price and income elasticities.

The energy crisis results from the fact that the world is rapidly running out of oil and that other energy sources seem unlikely to fill the gap. Without strong energy policies now to set energy priorities and facilities development of new energy technology, we may in fact face a genuine shortage of energy supplies in the late 1980s or 1990s. If the energy capacity shortage arises, it will not be easily dispelled.

The economic model specified in the methodology section of this paper contains four independent variables: price, income, heating degree days, and cooling degree days. Numerous explanatory variables could have been used to explain the effects of demand for energy in minority households. For example, stock appliances, structure type (i.e., single family vs. multifamily), house age, and space heating fuel type are just a few of the other variables that could have been estimated in the model. However, four variables were used for simplicity.

The results of the price and income elasticities generated from OLS estimation of each equation suggest that policymakers could find it difficult to provide the appropriate solutions for energy needs in minority households. The wide variations in price and income elasticities illustrated in Table 7 exemplify that extreme care should be used when selecting a functional form for estimation.

The appendix of this paper discusses the Ramsey and Zarembka tests used to eliminate problems such as omitted variables, multicollinearity, identification problems, and aggregation problems, that arise sometimes in estimating economic models. The tests are just one way of providing solutions in estimating equation for a better link in price and income elasticities.

### Conclusions

Based on the literature review and the models estimated in this study, the following conclusions are adduced:

- 1) All price estimates confirm initial hypothesis. (There is an inverse relationship between price and quantity.)
- 2) Income elasticities are all positive and inelastic.
- 3) The HDD and CDD coefficients are both positive and significant in terms of t-ratios and are inelastic. Thus weather variables do affect the quantity of energy demanded.

The apparatus of economic theory, to provide sound conclusion, requires clarity in one's assumptions, consistency in one's hypotheses, rigor in one's analysis and aids immeasurably in providing a higher plan for policy formulation.

Focus for Further Research

The supply relation in the market for energy sources needs to be investigated. The oil crises of 1973 was as much a supply problem as it was a demand problem.

While no evidence has been presented so far for neglecting that dimension to the energy problem, future studies should then consider seriously that aspect, especially with the precarious political and economic conditions of the Middle East.

## APPENDIX

### Actual Test Performance

This discussion is extracted from the Appendix in Ramsey and Zarembka's paper.<sup>1</sup> The earlier paper by Ramsey is extensively reviewed in the first part of this Appendix. The discussion above is a much more explicit statement on how actually to perform the test. The BLUS residual vector  $U$  (best, linear, unbiased scalar covariance matrix) is a  $(N - K) \times 1$  vector, which has the following properties:

- 1)  $E(\tilde{U}) = 0$
- 2)  $E(\tilde{U}\tilde{U}) = \sigma^2 I_{n-k}$

This contrasts with the specification of the error term under the "full ideal conditions" where we assumed:

$$U \sim N(0, \sigma^2 I_n)$$

### RESET

RESET stands for regression specification error test. This test is performed by using  $U$ , the BLUS residual, on the vectors  $j$ ,  $j = 1, 2, \dots, k$ . ( $\hat{\epsilon}_j$  is the predicted value of the  $i$ th observation on the dependent variable). It is calculated by multiplying the set of BLUS

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<sup>1</sup>Ramsey and Zarembka, p. 476.

residuals and the predicted  $y$ . An F-statistic defined by the ratio of the regression sum of squares to the error sum of squares. Reject the null hypothesis of no specification error for large values of F at a selected significance level.

### RASET

RASET is the Spearman's Rank Correlation test on the correlation between the ranking of  $\tilde{U}_i^2$  and  $\xi_{1i}$ . The alternative hypothesis is that each element of the mean vector of  $\tilde{U}$  is a monotonic function of the corresponding element of the mean vector of  $\xi_i$ . The population correlation is non-zero. The test statistic is distributed as student "t" with  $(N - K - 2)$  degrees of freedom.

### BAMSET

BAMSET is the modification of Bartlett's "M" test for heterogeneity of variances. It is therefore used to test for heteroscedasticity. Under the null hypothesis, the covariance matrix of the BLUS vector  $\tilde{U}$  is  $\sigma^2 I_{n-k}$  and under the alternative, the covariance matrix is assumed to be a diagonal matrix with variances  $\sigma_1^2, \sigma_2^2 \dots \sigma_{n-k}^2$  on the diagonal.

The statistic is a maximum Likelihood Ratio test.

$$L^* = \pi_1^k = 1 (s_i^2/s^2) v_i/2$$

where,

$k$  = Number of subgroups of squared residuals

$$S_j^2 (1/v_j) \text{ each } v_j \text{ is an } \sum_{j=1}^k v_j \bar{U}_j^2,$$

integer approximately equal to:

$$(N - k)/3.0, \sum_{j=1}^k v_j = v = (N - k)$$

$$S = (1/v) \sum_{j=1}^k v_j \bar{U}_j^2.$$

The authors set  $k = 3$ , i.e., three groups. Actual test statistics are  $M = -2 \ln 1^*$  which is distributed as central Chi-square ( $\chi^2$ ) with  $(k - 1)$  degrees of freedom.

#### KOMSET

KOMSET is the Kolmogorov test on the cumulative distribution of the variables  $W_r$ ,  $r = 1, 2, \dots, (N - k)/2$ , where,

$$W_r = \bar{U}_{2i}^2 / \bar{U}_{2i-1}^2, r, i = 1, 2, \dots, (N - k)/2$$

Under the null hypothesis,  $W_r$  is distributed  $F_1, 1$ . Under the alternative of omitted variable of specification errors,  $W_r$  is distributed approximately as  $A, F, b_1, b_2$ , where,  $A$  is a scale factor and the degrees of freedom  $b_1, b_2$  are both greater than one.

#### Chi-Square (Goodness of Fit Test)

This is used to test the assumption of normality for the distribution of the BLUS residual  $\tilde{U}_i$ ,  $i = 1, 2, \dots, (N - k)$ . Under the null hypothesis each  $\tilde{U}_i$  is distributed  $N(0, \sigma^2)$ . The alternative hypothesis that  $U_i$  is not distributed normally but with non-zero mean.



The test is influenced by the value of  $n^2$  which is unknown. Maximum likelihood estimator of  $n^2$  is determined as:

$$s^2 = (1/N - k) \sum_{i=1}^N N - k (\tilde{U}_i)^2$$

This equation is used to calculate the probabilities under the null hypothesis. The test statistic is:

$$\chi^2 = 1/N \sum_{i=1}^k n_i^2 / P_{oi-n},$$

where,

$N$  = The number of residuals;

$k$  = The number of intervals;

$n_i$  = The number of residuals in  $i^{\text{th}}$  intervals; and

$P_{oi}$  = The probability under the null hypothesis of a residual lying in the  $i^{\text{th}}$  interval.

Heckman and Polachek<sup>2</sup> applied the selection of function form methods to a study of the relationship between earnings and schooling. The author's method of analysis is quite different from the method used by Ramsey and Zarembka. The latter simply identified several popular functions and then imposed various statistical tests to determine the appropriate functional form. Heckman and Polachek, on the other hand, did not rely on the established functional relationships explaining earnings and schooling. Rather, the authors used a flexible specification of a functional form and allowed the data to select a specific

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<sup>2</sup>Heckman and Polachek, pp. 350-354.

appropriate form. They pointed out, "in this article we take an agnostic view. We accept the intuitively plausible argument that earnings are related to education and to the extent of market experience, and ask the data to give information on the correct functional form."<sup>3</sup> Like Ramsey and Zarembka, Heckman and Polachek concluded that a comparison of  $R^2$  is inappropriate since different dependent variables would be involved in the comparison.

### Statistical Tests

A "good" statistical test is:

- 1) One for which the probability of rejecting the null (or power of the test) is less than or equal to the chosen significance level when the null is true (low probability of a type of one error);
- 2) One for which the power is large when the null is false (low probability of a type II error); and
- 3) One which is insensitive to departures from the assumptions of the model (a robust test).<sup>3</sup>

The test for robustness in a model is especially relevant in energy demand studies where various specifications of the same idea have been offered and also where alternative functional forms may be utilized. It becomes critical as to which specifications of the energy demand models is true in a probability sense. In this appendix the various

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<sup>2</sup>Ibid., pp. 350-354.

<sup>3</sup>J. G. Thursby, "A Test Strategy for Discriminating between Autocorrelation and Misspecification in Regression Analysis," Review of Economic Statistics vol. 63, no. 1 (1981):117-123.

tests used in this study are explained. These tests are based on Ramsey's seminar paper.

### Tests for Specification Errors in Classical Linear Least-Squares Regression

The tests developed by Ramsey are based on the assumption that if the model is correctly specified, our best estimates of the disturbance should exhibit properties which would not lead to a rejection of the full ideal conditions of the classical linear model. The specification errors that are considered include:

- 1) The case of omitted variables;
- 2) Incorrect functional form; and
- 3) Heteroscedasticity.

We begin by defining a correctly defined energy demand model of the form:

$$Y = x\beta + u \text{ where,}$$

$$y = N \times 1 \text{ vector} = \text{Dependent variable}$$

$$x = N \times k \text{ vector} = \text{Independent variable with rank} = k < n;$$

$$\beta = k \times 1 \text{ vector} = \text{Vector of coefficients; and}$$

$$u = n \times 1 \text{ vector} = \text{Vector of disturbances satisfying} \\ u \sim n(0, \sigma^2 I_n).$$

The least-squares residual vector is then defined as:

$$\hat{u} = y - x\hat{\beta}$$

$$= y - x (x'x)^{-1} x'y$$

$$= [I - x (x'x)^{-1} x'] y$$

$$= My \text{ where } M = I - x (x'x)^{-1} x'$$

### Properties of M

- 1) Idempotent
- 2) Positive semi-definite
- 3) Non-diagonal
- 4) Rank = n - k

### Proofs

#### Proof 1:

Definition: A is idempotent if  $A^1 = A = A^2$ .

Let:  $x (x'x)^{-1} x' = A$  so that  $A^1 [x (x'x)^{-1} x']^1 = x (x'x)^{-1} x'$ .

Claim: Then  $I - A$  is also idempotent

Look at  $(I - A)^2 = I - 2A + A^2 = I - A$ . Alternatively,

$$\begin{aligned} [I - x (x'x)^{-1} x']^2 &= I - x (x'x)^{-1} x' - x (x'x)^{-1} x' \\ &+ x (x'x)^{-1} x' x (x'x)^{-1} x' = I - x (x'x)^{-1} x' \end{aligned}$$

#### Proof 2:

Definition: The quadratic form  $x'Ax$  and the associated symmetric matrix A are said to be positive semi-definite if  $x'Ax \geq 0$  holds for any x.

So,  $A'A = A$  in view of  $A' = A = A^2$ . Any quadratic form  $x'Ax = (Ax)^1 Ax$  = sum of the squares of the elements of Ax and is nonnegative.

#### Proof 3:

To show rank = n - k, we use the following: rank of an idempotent matrix is equal to its trace:

$$\begin{aligned} \text{tr } M &= \text{tr } I - \text{tr } x (x'x)^{-1} x' \\ &= n - \text{tr } (x'x)^{-1} x'x \\ &= n - k = \text{rank } m. \end{aligned}$$

Proof 4:

Here, all we need to show is that the M matrix is nonscalar since a scalar matrix is only a special form of a diagonal matrix, i.e., if it is nonscalar then it is non-diagonal.

$$m = I - x (x'x)^{-1} x' = I - xx^{-1} (x')^{-1} x' \neq 0$$

the zero matrix being a scalar matrix. Since  $u = m (x + u) = mu$ , the covariance matrix of the LS residual is:

$$\begin{aligned} \Sigma (\hat{u}\hat{u}') &= m (\hat{u}\hat{u}') m' = \sigma^2 mm' \\ &= \sigma^2 m \end{aligned}$$

Recalling the properties of the M matrix, we reach the following conclusion which serves as the motivation for the tests we will be discussing, that is, conclude that, even though the disturbance terms  $u_i$  ( $i = 1, \dots, N$ ) are independent and identically distributed as  $N(0, \sigma^2)$ , the LS residuals  $\hat{u}_i$  are not independently and identically distributed. Put in other words, homoscedastic and uncorrelated disturbances do not guarantee homoscedastic and uncorrelated LS residuals. Let us derive the distribution of  $(\hat{u}'\hat{u})/\sigma^2$ . Simply call  $\hat{u}\hat{u}' \rightarrow n \hat{\sigma}^2$  so that we are going to deduce distribution of a quadratic form. Model is  $y = x\beta + u$

Rank  $(x) = k$ ,  $u \sim N(0, \sigma^2 I_n)$ .

Now,

$$\hat{\sigma}^2 = \frac{1}{n} y' [I - x (x'x)^{-1} x'] y$$

since  $\hat{u} = My$   $xn \div 2$  so,

$$\frac{n \hat{\sigma}^2}{2} = \frac{y' [I - x (x'x)^{-1} x'] y}{\sigma^2}$$

$y \sim N(x\beta, \sigma^2 I_n)$ .

therefore,

$$\frac{n \hat{\sigma}^2}{\sigma^2} = y^{*'} [I - x (x'x)^{-1} x'] y^*$$

Recall:  $I - x (x'x)^{-1} x'$  is idempotent with rank  $n - k$ .

So that,

$$\frac{n \hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-k} \text{ and } \lambda^2 = \beta' x' [I - x (x'x)^{-1} x'] x \beta = 0$$

Conclude that the distribution of  $(\hat{u}^1 \hat{u})/\sigma^2$  is central  $\chi^2$  with  $N - K$  degrees of freedom.

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